Ann. Sc. Norm. Super. Pisa Cl. Sci. (5)
Vol. XVI (2016), 97-128

Intrinsic co-local weak derivatives and Sobolev spaces between manifolds

ALEXANDRA CONVENT AND JEAN VAN SCHAFTINGEN

Abstract. We define the notion of co-locally weakly differentiable maps from a manifold M to a manifold N. If $p \geq 1$ and if the manifolds M and N are endowed with a Riemannian metric, this allows us to define intrinsically the homogeneous Sobolev space $\dot{W}^{1,p}(M, N)$. This new definition is equivalent to the definition by embedding in the Euclidean space and to that of Sobolev map into a metric space. The co-local weak derivative is an approximate derivative. The co-local weak differentiability is stable under a suitable weak convergence. The Sobolev spaces can be endowed with various intrinsic distances that induce the same topology and for which the space is complete.

Mathematics Subject Classification (2010): 58D15 (primary); 46E35, 46T10, 53C25, 58E20 (secondary).